## Student's Interval Notation Notes with Practice

Interval notation is another method for writing domain and range.
In set builder notation braces (curly parentheses \{ \} ) and variables are used to express the domain and range. Interval notation is often considered more efficient.

In interval notation, there are only 5 symbols to know:

- Open parentheses ()
- Closed parentheses [ ]
- Infinity <apply>oo</apply>
- Negative Infinity $-\infty$
- Union Sign U

To use interval notation:
Use the open parentheses ( ) if the value is not included in the graph. (i.e. the graph is undefined at that point... there's a hole or asymptote, or a jump)

If the graph goes on forever to the left, the domain will start with ( $-\infty$. If the graph travels downward forever, the range will start with ( $-\infty$. Similarly, if the graph goes on forever at the right or up, end with <apply> $<$ </apply>)

Use the brackets [ ] if the value is part of the graph.
Whenever there is a break in the graph, write the interval up to the point. Then write another interval for the section of the graph after that part. Put a union sign between each interval to "join" them together.

Now for some practice so you can see if any of this makes sense.
Write the following using interval notation:

## Exercise 1



Figure 1

## Exercise 2



Figure 2

## Exercise 3



Figure 3

## Exercise 4



Figure 4

## Exercise 5



Figure 5

Exercise 6


Figure 6

Write the domain and range of the following in interval notation:

## Exercise 7



Figure 7

Exercise 8


Figure 8

## Exercise 9



Figure 9

Exercise 10


Figure 10

## Exercise 11



Figure 11
Exercise 12


Figure 12

## Exercise 13



Figure 13

## Exercise 14



Figure 14

## Exercise 15



Figure 15

## Exercise 16



Figure 16

## Exercise 17



Figure 17

## Exercise 18



Figure 18

## Exercise 19



Figure 19

Exercise 20


Figure 20

## Putting the Pieces Together

## Part 1: Training for a Race

Saundra is a personal trainer at a local gym. Earlier this year, three of her clients asked her to help them train for an upcoming 5K race. Though Saundra had never trained someone for a race, she developed plans for each of her clients that she believed would help them perform their best.

She wanted to see if her plans were effective, so when she attended the race to cheer them on, she collected data at regular intervals along the race. Her plan was to create graphs for each of the runners and compare their performances.

Since each had an individualized strategy, each runner ran a different plan during the race. One of her clients (Sue, the oldest one), was supposed to begin slowly, increasing over the first kilometer until she hit a speed which she believed she could maintain over the rest of the race.

Her second client, Jim, was supposed to begin with a strong burst for the first kilometer, then slow to a steady pace until the final kilometer when he would finish with a strong burst.

Her third client, Jason, is a very experienced runner. His plan was to run at a steady pace for the first two kilometers, then run at his maximum speed for the final 3 kilometers.

Each of the clients came close to performing as they planned.

1. Saundra created graphs for two of the clients, but she set them aside without labeling the graphs. Now she cannot remember whose graphs she has. Can you identify the client based on these graphs? Explain how you know.


Graph 1


Graph 2
2. Describe how the runner in Graph \#1 performed. For what distance did the runner increase speed, decrease speed, or maintain speed?
3. Compare the performance of the runner in Graph \#2 to the runner in Graph \#1.
4. Saundra found the data for her third client on her desk. Graph the data for this runner.

| Time | Km |
| :--- | :--- |
| $4: 00$ | 1 |
| $8: 30$ | 2 |
| $13: 00$ | 3 |
| $22: 00$ | 4 |
| $26: 00$ | 5 |



While you may be tempted to find a line that describes this data, a single line does not really show how the runner performed at each interval. A piecewise function is a graph that shows differences in specified intervals; that is, it is a graph with two or more pieces. The slope of the pieces may not be the same and even the shape of the pieces may not be the same.
5. Connect the points in the third graph to show the "pieces" of different performance levels by the runner.
6. Using the third graph, write the equations of the "pieces," or segments, of the graph. Be sure to indicate the appropriate interval for each piece (for which $x$-values that equation is the correct graph).

## Worksheet \#10-Interval Notation

The following are the Number Line Solutions for the Interval Notation and Inequality Forms on the other page. Practice going from each form to any other form. In particular, practice being able to interpret the Number Line Solution back into Inequality Form and Interval Notation.


Name $\qquad$ Date $\qquad$
Please describe the domain and range of each function using interval notation.
1.


Domain:

Range:
3.


Domain:

Range:
5.


Domain:

Range:
2.


Domain:

Range:
4.


Domain:

Range:
6.


Domain:

Range:
7.


Domain:
Range:
9.


Domain:
Range:
11.


Domain:

Range:
8.


Domain:
Range:
10.


Domain:
Range:
12.


Domain:

Range:
13.


Domain:

Range:
14.


Domain:
Range:

Name $\qquad$ Date $\qquad$ Period $\qquad$
Interval Notation

Please write the following sets in interval notation.

1. The set of all numbers less than or equal to -3 .
2. The set of all real numbers greater than or equal to 4 and less than 8 .
3. The set of all real numbers either greater than 6 or between, but not equal to, -3 and -2 .
4. The set of all real numbers between 12 and 8 , including 12 but not including 8.

Display the following sets on real number lines.
5. $[-3,1)$

6. $(2,1)$

7. $(2,4]$ and $[3,8)$

8. $(-1,-3)[(1,2]$


## Putting the Pieces Together

## Part 4: Manufacturing Moldings

Piecewise functions do not always have to be line segments. The "pieces" could be pieces of any kind of graph. Try to graph some of these piecewise functions. You may find it helpful to use what you already know about transformations of the parent functions

1. $f(x)=\left\{\begin{array}{l}x^{2}+4, x<0 \\ \sqrt{x}+4, x \geq 0\end{array}\right.$

2. $f(x)=\left\{\begin{array}{l}|x|-1, x>-1 \\ x+3, x \leq-1\end{array}\right.$


3. $f(x)=\left\{\begin{array}{l}3, \text { for }-3 \leq x<-1 \\ x^{2}, \text { for }-1 \leq x<1 \\ 3, \text { for } 1 \leq x \leq 3\end{array}\right.$
4. In some manufacturing settings, machines can be programmed to make certain cuts based on piecewise functions the operator can define. What equations would you program into the machine to cut to create the following shape?


Name $\qquad$ Date $\qquad$ Period $\qquad$
Graph the piecewise function.

| 1) $f(x)= \begin{cases}3 x & \text { if } x \neq 0 \\ 4 & \text { if } x=0\end{cases}$ |  | 6) $f(x)= \begin{cases}1+x & \text { if } x<0 \\ x^{2} & \text { if } x \geq 0\end{cases}$ |
| :---: | :---: | :---: |
| 2) $f(x)=\left\{\begin{array}{l}-2 x+3 \text { if } x<1 \\ 3 x-2 \text { if } x \geq 1\end{array}\right.$ |  | 7) $f(x)= \begin{cases}\frac{1}{x} & \text { if } x<0 \\ \sqrt{x} & \text { if } x \geq 0\end{cases}$ |
| 3) $f(x)=\left\{\begin{array}{lr}x+3 & \text { if } x<-2 \\ -2 x-3 & \text { if } x \geq-2\end{array}\right.$ |  | 8) $f(x)= \begin{cases}\|x\| & \text { if }-2 \leq x<0 \\ 1 & \text { if } x=0 \\ x^{3} & \text { if } x>0\end{cases}$ |
| 4) $f(x)= \begin{cases}x+3 & \text { if }-2 \leq x<1 \\ 5 & \text { if } x=1 \\ -x+2 & \text { if } x>1\end{cases}$ |  | 9) $f(x)= \begin{cases}3+x & \text { if }-3 \leq x<0 \\ 3 & \text { if } x=0 \\ \sqrt{x} & \text { if } x>0\end{cases}$ |
| 5) $f(x)= \begin{cases}2 x+5 & \text { if }-3 \leq x<0 \\ -3 & \text { if } x=0 \\ -5 x & \text { if } x>0\end{cases}$ |  |  |

Graph each piecewise function.
1.

$$
f(x)=\left\{\begin{array}{l}
x+1 \text { f0 } \leq x<5 \\
2 x-4 \text { if } 5 \leq x<10
\end{array}\right.
$$

2. 

$$
g(x)=\left\{\begin{array}{l}
3 x-4 \text { fi } \leq x<6 \\
20-x \text { if } 6 \leq x 12
\end{array}\right.
$$

3. 

$$
m(x)= \begin{cases}20 & \text { if } 0=x<10 \\ \frac{x}{2}+15 & \text { f } 10=x<20\end{cases}
$$




4.

$$
f(x)= \begin{cases}4 x & \text { if } 0=x<2 \\ -2 x+10 & \text { if } 2=x<5 \\ 2 & \text { if } 5=x<10\end{cases}
$$

5. 

$$
h(x)=\left\{\begin{array}{cl}
-2 & \text { if } x<0 \\
x+1 & \text { if } 0=x=10 \\
-1 / 2 x+16 & \text { if } x>10
\end{array}\right.
$$

6. 

$$
b(x)=\left\{\begin{array}{cl}
2 & \text { if } x<1 \\
2 x & \text { if } 1=x<3 \\
7-\frac{1}{3} x & \text { if } x>3
\end{array}\right.
$$


7.

$$
k(x)= \begin{cases}2 x+3 & \text { if } x<4 \\ x-1 & \text { if } 4=x=9\end{cases}
$$



$$
g(x)= \begin{cases}5-x & \text { if } x<2 \\ x-1 & \text { if } 2=x=3\end{cases}
$$

Please write the piecewise function represented by each graph.

10.

$\qquad$

$-x-5$ for $-5 \leq x \leq-2$
$f(x)=-x^{2}+1$ for $-2<x<2$
$(x-3)^{3}+2$ for $2 \leq x \leq 4$

Find the following values:
$f(-4)=$ $\qquad$

$$
f(-2)=
$$

$\qquad$ $f(0)=$ $\qquad$
$\qquad$

What are the $x$-intercept(s) (zeroes) of the function? $\qquad$

What are the $y$-intercept(s) of the function? $\qquad$

## Extrema:

What is the maximum? $\qquad$ The minimum? $\qquad$
Give answers in interval notation for the next three questions.
Find the interval(s) on which the function is increasing. $\qquad$

Find the interval(s) on which the function is decreasing. $\qquad$

Find the interval(s) on which the function is constant. $\qquad$

List any points of discontinuity. $\qquad$
What is the rate of change on the interval $[-5,-2]$ ? $\qquad$

Graph the following piecewise function and then answer questions relating to it.
$x+1$ for $x<-4$
$f(x)=\begin{array}{ll}2 & \text { for }-4 \leq x<0 \\ x^{2} & \text { for } x \geq 0\end{array}$


Find the following values:
$f(-6)=$ $\qquad$
$f(-4)=$ $\qquad$
$f(0)=$ $\qquad$
$f(3)=$ $\qquad$

What are the x-intercept(s) (zeroes) of the function? $\qquad$
$\qquad$
What are the $y$-intercept(s) of the function? $\qquad$
-

## Extrema:

What is the maximum? $\qquad$ The minimum?

Give answers in interval notation for the next three questions.
Find the interval(s) on which the function is increasing. $\qquad$

Find the interval(s) on which the function is decreasing. $\qquad$

Find the interval(s) on which the function is constant.
$\qquad$

List any points of discontinuity. $\qquad$
What is the rate of change on the interval $[-6,-4) ?$ $\qquad$

## Piecewise Functions Quiz

Please use the function below to answer the questions that follow. Please use interval notation to communicate all intervals.

$$
f(x)= \begin{cases}-x+2 & \text { if } x<2 \\ 2 x+2 & \text { if } x>2\end{cases}
$$

1. What is the domain of the function in interval notation?
2. What is the range of the function in interval notation?
3. Are there any points of discontinuity? If so, where are they located?
4. Is there a maximum value? If so, what is it?
5. Is there a minimum value? If so, what is it?
6. What is the interval of decrease?
7. What is the interval of increase?


Please use the graph below to answer the questions that follow. Use interval notation when the answer is an interval.
8. What is the constant interval?
9. Over what interval is the function decreasing?
10. Over what interval is the function increasing?
11. Are there any points of discontinuity? If so, what are they?
12. What are the domain and range of the function shown on the graph?
13. Is there a maximum value? If so, where does it occur?
14. Is there a minimum value? If so, where does it occur?
15. Please write a piecewise function for the graph.
3.

$$
g(x)= \begin{cases}5 x-1 & \text { if } x<2 \\ x-9 & \text { if } x \geq 2\end{cases}
$$

Domain:

Range:

Constant Interval:

Increasing Interval:


Decreasing Interval:

Maximum:

Minimum:

Points of Discontinuity:
4.

$$
f(x)=\left\{\begin{array}{cl}
3 x-4 & \text { if } x<-6 \\
-x-1 & \text { if }-6 \leq x<-1 \\
4 & \text { if } x \geq-1
\end{array}\right.
$$

Domain:

Range:

Constant Interval:

Increasing Interval:


Decreasing Interval:

Maximum:

Minimum:

Points of Discontinuity:

